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SUGGESTED SOLUTION

FYJC

SUBJECT- STATISTICS

Test Code – FYJ 6019

BRANCH - () (Date :)

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ANSWER : 1

(A) An unbiased coin is tossed 4 times.

$\therefore$ the total number of outcomes n(S) = 2^4 = 16

Let A = Event that exactly two heads are shown.

= {(HHTT), (HTHT), (HTTH), (THTH), (THHT), (TTHH)}

$\therefore$ favourable outcomes for the event A, n(A) = 6

Now, P(A) = $\frac{n(A)}{n(S)}$

= $\frac{6}{16}$

$\therefore$ P(A) = $\frac{6}{16}$

Hence, the probability that it shows head exactly two times is $\frac{6}{16}$.

(02)

(B) Random Experiment = One card is drawn at random from a pack of 52 cards.

$\therefore$ n(S) = ${}^{52}C_1$ = 52.

Let event A : Card drawn is King

and event B : Card drawn is Queen

Since pack of 52 cards contains, 4 king card from which any one king card can be drawn in ${}^{4}C_1$ = 4 ways.

$\therefore$ n(A) = 4

$\therefore$ P(A) = n(A)/n(S) = 4/52 = $\frac{1}{13}$

Similarly, a pack of 52 cards contains, 4 queen cards from which any one queen card can be drawn in ${}^{4}C_1$ = 4 ways.
$\therefore$ n(B) = 4

$\therefore$ P(B) = n(B) /n(S) = 4/52 = $\frac{1}{13}$

Since A and B are mutually exclusive events

$\therefore$ required probability P (King or queen)

= P (A $\cup$ B) = P(A) + P(B) = 4/52 + 4/52 = 2/13

(02)

(C) In a lot of 12 items, 4 items are defective.

$\therefore$ 8 items are non – defective.

Two items are drawn at random from the lot without replacement.

$\therefore$ total number of outcomes n(S) = ${}^{12}C_2 = \frac{12 \times 11}{2 \times 1}$ = 66

Let A = Event that both items are non – defective.

There are 8 non – defective items.

$\therefore$ favourable out comes for the event A is

n(A) = ${}^{8}C_2 = \frac{8 \times 7}{2 \times 1}$ = 28

Now, P(A) = $\frac{\mathrm{n(A)}}{\mathrm{n(S)}} = \frac{8_{C_2}}{12_{C_2}}$

$= \frac{28}{66} = \frac{14}{33}$

(02)

ANSWER : 2

(A) Two fair dice are thrown.

$\therefore$ total number of outcomes n = n(S) = 36.

Let A = Event that the sum of the numbers is at least 10.

B = Event that the sum of the numbers exceeds 7.

$A \cap B$ = Event that the sum of the numbers is at least 10 and it exceeds 7.

Now, A = [(4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)}

B = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5) (6, 4) ,(5, 6), (6, 5),(6, 6)}

$A \cap B$ = {(4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)}

$\therefore$ P(A) = $\frac{6}{36}$, P(B) = $\frac{15}{36}$, P($A \cap B$) = $\frac{6}{36}$

A | B = Event that the sum of numbers is at least 10 given that it exceeds 7.

$\therefore$ P(A | B) = $\frac{\mathrm{P}(A \cap B)}{\mathrm{P(B)}}$

$$\frac{\frac{6}{36}}{\frac{15}{36}} = \frac{6}{36} \times \frac{36}{15}$$

$\therefore$ P(A | B) = $\frac{6}{15} = \frac{2}{5}$

Hence, the probability that the sum of number is at least 10, given that it exceeds 7 is $\frac{6}{15} = \frac{2}{5}$.

(03)

(B) Let A be the event that student A can solve the problem.

B be the event that student B can solve problem.

C be the event that student C can solve problem.

$\therefore$ P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{4}$, P(C) = $\frac{1}{5}$

$\therefore P(A') = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(B') = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(C') = 1 - P(C) = 1 - \frac{1}{5} = \frac{4}{5}$

Since A, B, C are independent events

$\therefore$ A′, B′ C′ are also independent events

(i) Let X be the event that problem is solved.
Problem can be solved if at least one of the three students solves the problem.

P(X) = P (at least one student solves the problem)

= 1 – P (no students solved problem)

$= 1 - P(A' \cap B' \cap C')$

$= 1 - P(A')\, P(B')\, P(C')$

$= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = 1 - \frac{2}{5} = \frac{3}{5}$

(ii) Let Y be the event that problem is not solved

$\therefore P(Y) = P(A' \cap B' \cap C')$

$= P(A')\, P(B')\, P(C')$

$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$

(iii) Let Z be the event that exactly two students solve the problem.

$\therefore P(Z) = P(A \cap B \cap C') \cup P(A \cap B' \cap C) \cup P(A' \cap B \cap C)$

$= P(A) \cdot P(B) \cdot P(C') + P(A) \cdot P(B') \cdot P(C) + P(A') \cdot P(B) \cdot P(C)$

$= \left(\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5}\right) + \left(\frac{1}{3} \times \frac{3}{4} \times \frac{1}{5}\right) + \left(\frac{2}{3} \times \frac{1}{4} \times \frac{1}{5}\right)$

$= \frac{4}{60} + \frac{3}{60} + \frac{2}{60} = \frac{9}{60} = \frac{3}{20}$

(03)

ANSWER : 3

(A) Let event A : first ball drawn is black.

Event B : Second ball drawn is black.

$\therefore$ required probability = P(both are black balls)

= P (A $\cap$ B) = P(A) P(B/A)

Now P(A) = 4/10

Since first black ball is not replaced in the urn, therefore now we have 9 balls containing 3 black balls.

$\therefore$ Probability of getting second black ball under the condition that first black ball is not replaced in the pack = P(B/A) = 3/9

$\therefore$ P(both are black balls) = P (A $\cap$ B)

= P(A) P(B/A) = (4/10) $\times$ (3/9) = 2/15

The first ball can be black or non – black (white)

case 1 : The first ball is white : The probability of second ball is black is

$$\frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$$

case 2 : The first ball is black : The probability of second ball is black is

$$\frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$$

$\therefore$ The required probability

$$= \frac{4}{15} + \frac{2}{15} = \frac{6}{15} = \frac{2}{5}$$

(04)

(B) Since two dice are thrown,
therefore n (S) = 36

(i) Let event A : Sum of the numbers is divisible by 3

$\therefore$ Possible sums are 3, 6, 9, 12.

$\therefore$ A = {(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4),(6, 3), (6, 6)}

$\therefore$ n(A) = 12 $\therefore$ P(A) = n(A) /n(S) = 12/36

Let event B : sum of the numbers is divisible by 4.

$\therefore$ Possible sums are 4, 8, 12

$\therefore$ B = {(1, 3)(2, 2)(2, 6), (3, 1), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)}

$\therefore$ n(B) = 9 $\therefore$ P(B) = n(B) /n(S) = 9/36

$\therefore$ Event A $\cap$ B : Sum of the numbers is divisible by 3 and 4 i.e. divisible by 12.

$\therefore$ Possible sum is 12

$\therefore$ A $\cap$ B = {(6, 6)}

$\therefore$ n(A $\cap$ B) = 1

$\therefore$ P(A $\cap$ B) = n(A $\cap$ B)/ n(S) = 1/36

$\therefore$ Required probability

= P(Sum of the numbers is divisible by 3 or 4)

= $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

= 12/36 + 9/36 – 1/36 = 20/36 = 5/9

(ii) Let event X : Sum of the numbers is divisible by 3

$\therefore$ possible sums are 3, 6, 9, 12

$\therefore$ X = {(1, 2), (1, 5), (2, 1), (2, 4), (3, 3),(3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6)}

$\therefore$ n(X) = 12 $\therefore$ P(X) = n(X) /n(S) = 12/36

Let event Y : Sum of the numbers is divisible by 5.

$\therefore$ possible sums are 5, 10

$\therefore$ Y = {(1, 4), (2, 3), (3, 2), (4, 1), (4, 6), (5, 5), (6, 4)}

$\therefore$ n(Y) = 7 $\therefore$ P(Y) = n(Y) / n(S) = 7/36

$\therefore$ Event $X \cap Y$: Sum is divisible by 3 and 5

$\therefore$ $X \cap Y$ { } = ϕ

[X and Y are mutually exclusive events]

$\therefore$ $P(X \cap Y) = n(X \cap Y) / n(S) = 0$

$\therefore$ required probability = P (Sum of the numbers is neither divisible by 3 nor 5)

= $P(X' \cap Y') = P(X \cup Y)' = 1 - P(X \cup Y)$

= 1 – [P(X) + P(Y)]

= 1 – 19/36 = $\frac{17}{36}$

(04)